

Investigation

1

Making Comparisons

Surveys may report people's preferences in food, cars, or political candidates. Often, the favorites are easy to recognize. Explaining how much more popular one choice is than another can be more difficult. In this investigation, you will explore strategies for comparing numbers in accurate and useful ways. As you work on the problems, notice how the different ways of making comparisons send different messages about the numbers being compared.

1.1

Ads That Sell

An ad for the soft drink Bolda Cola starts like this:



To complete the ad, the Bolda Cola company plans to report the results of taste tests. A copywriter for the ad department has proposed four possible conclusions.

1. In a taste test, people who preferred Bolda Cola outnumbered those who preferred Cola Nola by a ratio of 17,139 to 11,426.
2. In a taste test, 5,713 more people preferred Bolda Cola.
3. In a taste test, 60% of the people preferred Bolda Cola.
4. In a taste test, people who preferred Bolda Cola outnumbered those who preferred Cola Nola by a ratio of 3 to 2.

Problem 1.1 Exploring Ratios and Rates

- A. Describe what you think each statement above means.
- B. Which of the proposed statements do you think would be most effective in advertising Bolda Cola? Why?
- C. Is it possible that all four statements are based on the same survey data? Explain your reasoning.
- D. In what other ways can you express the claims in the four proposed advertising statements? Explain.
- E. If you were to survey 1,000 cola drinkers, what numbers of Bolda Cola and Cola Nola drinkers would you expect? Explain.

ACE Homework starts on page 10.



1.2 Targeting an Audience

Some middle and high school students earn money by delivering papers, mowing lawns, or baby-sitting. Students with money to spend are a target audience for some radio and television ads. Companies gather information about how much students watch television or listen to the radio. This information influences how they spend their advertising dollars.

As you work on this problem and the rest of the unit, you will see statements about ratio comparisons. In mathematics, it is acceptable to write ratios in different ways. Each way is useful.

Ways to Write a Ratio		
3 to 2	3 : 2	$\frac{3}{2}$

It can be confusing to see a fraction representing a ratio. A ratio is usually, but not always, a *part-to-part* comparison. A fraction usually means a *part-to-whole* comparison. The context can help you decide whether a fraction represents a ratio.

Problem 1.2 Analyzing Comparison Statements

Students at Neilson Middle School are asked if they prefer watching television or listening to the radio. Of 150 students, 100 prefer television and 50 prefer radio.

- A.** How would you compare student preferences for radio or television?
- B.** Decide if each statement accurately reports results of the Neilson Middle School survey.
1. At Neilson Middle School, $\frac{1}{3}$ of the students prefer radio to television.
 2. Students prefer television to radio by a ratio of 2 to 1.
 3. The ratio of students who prefer radio to television is 1 to 2.
 4. The number of students who prefer television is 50 more than the number of students who prefer radio.
 5. The number of students who prefer television is two times the number who prefer radio.
 6. 50% of the students prefer radio to television.
- C.** Compare statements in parts (4) and (5) above. Which is more informative? Explain.
- D.** Consider only the accurate statements in Question B.
1. Which statement would best convince merchants to place ads on radio? Why?
 2. Which statement would best convince merchants to place ads on television? Why?

ACE Homework starts on page 10.

1.3

American Records

People are amazed and amused by records like the highest mountain, the longest fingernails, or the most spoons balanced on a face. What you have learned so far can help you make comparisons. In Problem 1.3, you will compare the largest living trees of different species.

Did You Know?

The champion white “Wye” oak tree near Wye Mills, Maryland, was about 460 years old when it fell during a thunderstorm in 2002. When the tree fell, thousands came by to gawk, shed tears, and pick up a leaf or a twig. Maryland officials carefully gathered and stored as much of the tree as they could until a suitable use could be found.

The challenge to find a white oak bigger than the Wye Mills tree launched the National Register of Big Trees. The search led to the discovery of a new national champion white oak in Virginia.



Go Online For: Information about big trees
PHSchool.com Web Code: ane-9031

You can describe the size of a tree by comparing it to other trees or familiar things.

Selected Champion Trees

Tree Type	Circumference (ft)	Height (ft)	Spread/Diameter (ft)
Giant Sequoia (Calif.)	83.2	275	107
Coast Redwood (Calif.)	79.2	321	80
Swamp Chestnut Oak (Tenn.)	23.0	105	216
Florida Crossopetalum (Fla.)	0.4	11	3
White Oak (Md.)	31.8	96	119

Source: *Washington Post*

Problem 1.3 Writing Comparison Statements

A. Use the table on the previous page.

1. How many coast redwood spreads does it take to equal the spread of the white oak?
2. Kenning says that the spread of the white oak is greater than that of the coast redwood by a ratio of about 3 to 2. Is he correct? Explain.
3. Mary says the difference between the heights of the coast redwood and the giant sequoia is 46 feet. Is she correct? Explain.
4. How many giant sequoia spreads does it take to equal the spread of the swamp chestnut oak?
5. Jaime says the spread of the giant sequoia is less than 50% of the spread of the swamp chestnut oak. Is he correct?
6. Len says the circumference of the swamp chestnut oak is about three fourths the circumference of the white oak. Is he correct?



B. The tallest person in history, according to the *Guinness Book of World Records*, was Robert Wadlow. He was nearly 9 feet tall. Write two statements comparing Wadlow to the trees in the table. Use fractions, ratios, percents, or differences.

C. Average waist, height, and arm-span measurements for a small group of adult men are given.

Waist = 32 inches Height = 72 inches Arm Span = 73 inches

Write two statements comparing the data on these men to the trees in the table. Use fractions, ratios, percents, or differences.

D. When a problem requires comparison of counts or measurements, how do you decide whether to use differences, ratios, fractions, or percents?

ACE Homework starts on page 10.

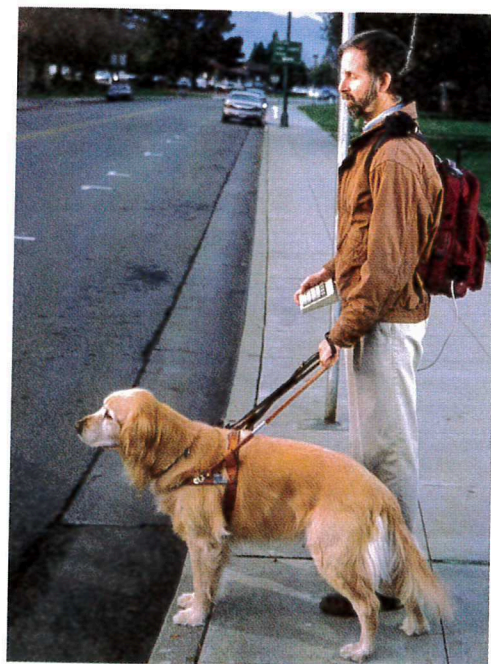
Applications

1. In a comparison taste test of two drinks, 780 students preferred Berry Blast. Only 220 students preferred Melon Splash. Complete each statement.
 - a. There were \blacksquare more people who preferred Berry Blast.
 - b. In the taste test, $\blacksquare\%$ of the people preferred Berry Blast.
 - c. People who preferred Berry Blast outnumbered those who preferred Melon Splash by a ratio of \blacksquare to \blacksquare .

2. In a comparison taste test of new ice creams invented at Moo University, 750 freshmen preferred Cranberry Bog ice cream while 1,250 freshmen preferred Coconut Orange ice cream.

Complete each statement.

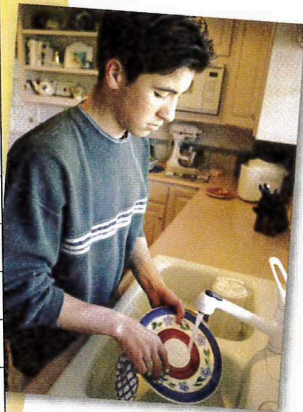
- a. The fraction of freshmen who preferred Cranberry Bog is \blacksquare .
 - b. The percent of freshmen who preferred Coconut Orange is $\blacksquare\%$.
 - c. Freshmen who preferred Coconut Orange outnumbered those who preferred Cranberry Bog by a ratio of \blacksquare to \blacksquare .
3. A town considers whether to put in curbs along the streets. The ratio of people who support putting in curbs to those who oppose it is 2 to 5.
 - a. What fraction of the people *oppose* putting in curbs?
 - b. If 210 people in the town are surveyed, how many do you expect to *favor* putting in curbs?
 - c. What percent of the people oppose putting in curbs?



Students at a middle school are asked to record how they spend their time from midnight on Friday to midnight on Sunday. Carlos records his data in the table below. Use the table for Exercises 4–7.

Weekend Activities

Activity	Number of Hours
Sleeping	18
Eating	2.5
Recreation	8
Talking on the Phone	2
Watching Television	6
Doing Chores or Homework	2
Other	9.5



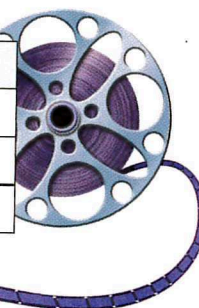
4. How would you compare how Carlos spent his time on various activities over the weekend? Explain.
5. Decide if each statement is an accurate description of how Carlos spent his time that weekend.
 - a. He spent one sixth of his time watching television.
 - b. The ratio of hours spent watching television to hours spent doing chores or homework is 3 to 1.
 - c. Recreation, talking on the phone, and watching television took about 33% of his time.
 - d. Time spent doing chores or homework was only 20% of the time spent watching television.
 - e. Sleeping, eating, and “other” activities took up 12 hours more than all other activities combined.
6. Estimate what the numbers of hours would be in *your* weekend activity table. Then write a ratio statement like statement (b) to fit your data.
7. Write other accurate statements comparing Carlos’s use of weekend time for various activities. Use each concept at least once.

<ol style="list-style-type: none"> a. ratio c. fraction 	<ol style="list-style-type: none"> b. difference d. percent
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8. A class at Middlebury Middle School collected data on the kinds of movies students prefer. Complete each statement using the table.

Types of Movies Preferred by Middlebury Students

Type of Movie	Seventh-Graders	Eighth-Graders
Action	75	90
Comedy	105	150
Total	180	240



- The ratio of seventh-graders who prefer comedies to eighth-graders who prefer comedies is ■ to ■.
- The fraction of total students (both seventh- and eighth-graders) who prefer action movies is ■.
- The fraction of seventh-graders who prefer action movies is ■.
- The percent of total students who prefer comedies is ■.
- The percent of eighth-graders who prefer action movies is ■.
- Grade ■ has the greater percent of students who prefer action movies.

Homework Help Online
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For: Help with Exercise 8
Web Code: ane-3108

9. Use the table.

Selected Champion Trees

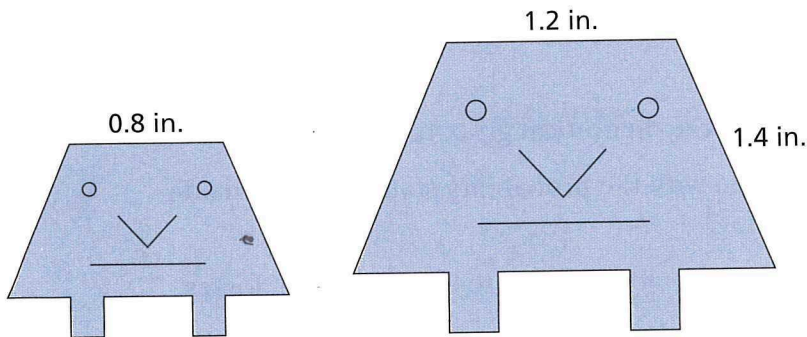
Tree Type	Height (ft)	Spread (ft)
Florida Crossopetalum	11	3
White Oak	96	119

- The height of the crossopetalum (kroh soh PET uh lum) is what fraction of the height of the white oak?
 - The height of the crossopetalum is what percent of the height of the white oak?
 - The spread of the crossopetalum is what fraction of the spread of the white oak?
 - The spread of the crossopetalum is what percent of the spread of the white oak?
10. In a survey, 100 students were asked if they prefer watching television or listening to the radio. The results show that 60 students prefer watching television while 40 prefer listening to the radio. Use each concept at least once to express student preferences.
- ratio
 - percent
 - fraction
 - difference

Connections

- 11.** A fruit bar is 5 inches long. The bar will be split into two pieces. For each situation, find the lengths of the two pieces.
- One piece is $\frac{3}{10}$ of the whole bar.
 - One piece is 60% of the bar.
 - One piece is 1 inch longer than the other.
- 12.** Exercise 11 includes several numbers or quantities: 5 inches, 3, 10, 60%, and 1 inch. Determine whether each number or quantity refers to the whole, a part, or the difference between two parts.

The sketches below show two members of the Grump family. The figures are geometrically similar. Use the figures for Exercises 13–16.



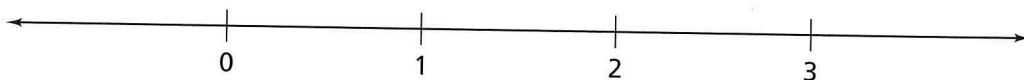
- 13.** Write statements comparing the lengths of corresponding segments in the two Grump drawings. Use each concept at least once.
- ratio
 - fraction
 - percent
 - scale factor
- 14.** Write statements comparing the areas of the two Grump drawings. Use each concept at least once.
- ratio
 - fraction
 - percent
 - scale factor
- 15.** How long is the segment in the smaller Grump that corresponds to the 1.4-inch segment in the larger Grump?
- 16. Multiple Choice** The mouth of the smaller Grump is 0.6 inches wide. How wide is the mouth of the larger Grump?
- A.** 0.4 in. **B.** 0.9 in. **C.** 1 in. **D.** 1.2 in.

The drawing below shows the Big Wheel spinner used in a game at the Waverly School Fun Night. It costs 20 cents to spin the wheel, and winners receive \$1.00. The chart shows the data from 236 spins of the Big Wheel. Use the spinner and the chart for Exercises 17–21.



Win	Lose
46	190

17. The sectors of the spinner are identical in size. What is the measure in degrees of each central angle?
18. You play the game once. What is the theoretical probability that you win?
19. Do the results in the table agree with the probability statement you made in Exercise 18? Why or why not?
20. Write statements comparing the number of wins to the number of losses. Use each concept at least once.
 - a. ratio
 - b. percent
 - c. difference
21. Which comparison from Exercise 20 is the best way to convey probability information about this game? Explain.
22. Copy the number line below. Add labels for 0.25 , $\frac{6}{8}$, $1\frac{3}{4}$, and 1.3 .



23. Write two unequal fractions with different denominators. Which fraction is greater? Explain.
24. Write a fraction and a decimal so that the fraction is greater than the decimal. Explain.

Copy each pair of numbers in Exercises 25–33. Insert $<$, $>$, or $=$ to make a true statement.

25. $\frac{4}{5} \blacksquare \frac{11}{12}$

26. $\frac{14}{21} \blacksquare \frac{10}{15}$

27. $\frac{7}{9} \blacksquare \frac{3}{4}$

28. $2.5 \blacksquare 0.259$

29. $30.17 \blacksquare 30.018$

30. $0.006 \blacksquare 0.0060$

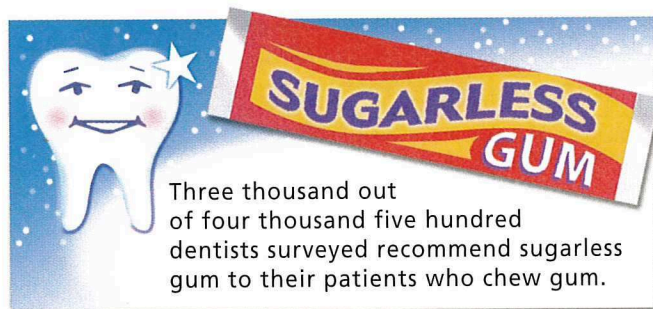
31. $0.45 \blacksquare \frac{9}{20}$

32. $1\frac{3}{4} \blacksquare 1.5$

33. $\frac{1}{4} \blacksquare 1.3$

Extensions

34. Rewrite this ad so that it will be more effective.



35. Use the table below.

Money Spent for Food

Where Food Is Eaten	1990	1998
Home	\$303,900,000,000	\$401,800,000,000
Away From Home	\$168,800,000,000	\$354,400,000,000

SOURCE: U.S. Census Bureau. Go to PHSchool.com for a data update. Web Code: ang-9041

- Compare money spent on food eaten at home and food eaten away from home to the total money spent for food. Write statements for each year.
- Explain how the statements you wrote in part (a) show the money spent for food away from home increasing or decreasing in relation to the total spent for food.

Use the table for Exercises 36–41.

Advertising Spending in the United States (millions)		
Placement	1990	2000
Newspapers	\$32,281	\$46,582
Magazines	\$6,803	\$11,096
Television	\$29,073	\$50,843
Radio	\$8,726	\$16,930
Yellow Pages	\$8,926	\$12,666
Internet	\$0	\$1,840
Direct Mail	\$23,370	\$41,601
Other	\$20,411	\$33,671
Total	\$129,590	\$215,229

SOURCE: U.S. Census Bureau. Go to PHSchool.com for a data update. Web code: ang-9041



36. Which placement has the greatest difference in advertising dollars between 1990 and 2000?
37. Find the percent of all advertising dollars spent on each placement in 1990.
38. Find the percent of all advertising dollars spent on each placement in 2000.
39. Use your results from Exercises 36–38. Write several sentences describing how advertising spending changed from 1990 to 2000.
40. Suppose you were thinking about investing in either a television station or a radio station. Which method of comparing advertising costs (differences or percents) makes television seem like the better investment? Which makes radio seem like the better investment?
41. Suppose you are a reporter writing an article about trends in advertising over time. Which method of comparison would you choose?

Mathematical Reflections

1

In this investigation, you explored several ways of comparing numbers. The problems were designed to help you understand and use different comparison strategies and recognize when each is most useful. The following questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Explain what you think each word means when it is used to make a comparison.
 - a. ratio
 - b. percent
 - c. fraction
 - d. difference

2. Give an example of a situation using each concept to compare two quantities.
 - a. ratio
 - b. percent
 - c. fraction
 - d. difference

