

2.1

Mixing Juice

Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times.

One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange-juice concentrate. To find the mix that tastes best, they decide to test some mixes.

Mix A		Mix B	
2 cups	3 cups	5 cups	9 cups
concentrate	cold water	concentrate	cold water

Mix C		Mix D	
1 cup	2 cups	3 cups	5 cups
concentrate	cold water	concentrate	cold water

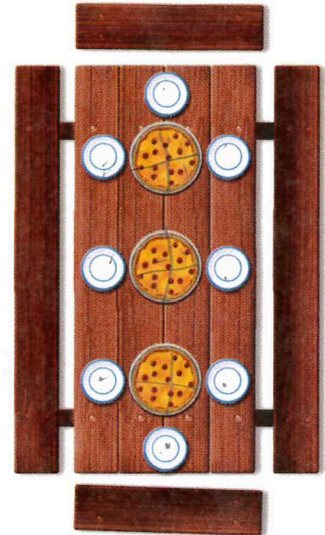
Problem 2.1 Developing Comparison Strategies

- A. Which mix will make juice that is the most “orangey”? Explain.
- B. Which mix will make juice that is the least “orangey”? Explain.
- C. Which comparison statement is correct? Explain.
 $\frac{5}{9}$ of Mix B is concentrate. $\frac{5}{14}$ of Mix B is concentrate.
- D. Assume that each camper will get $\frac{1}{2}$ cup of juice.
- For each mix, how many batches are needed to make juice for 240 campers?
 - For each mix, how much concentrate and how much water are needed to make juice for 240 campers?
- E. For each mix, how much concentrate and how much water are needed to make 1 cup of juice?

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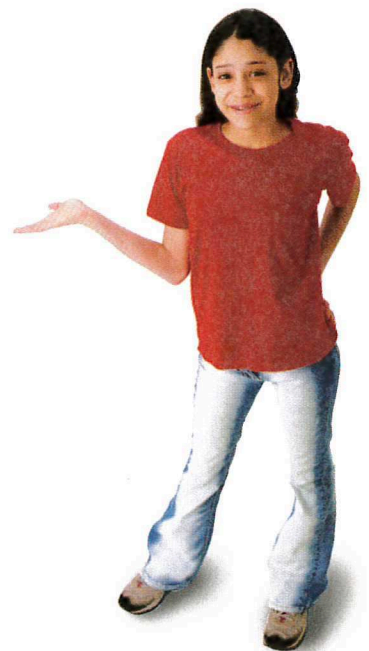
2.2 Sharing Pizza

The camp dining room has two kinds of tables. A large table seats ten people. A small table seats eight people. On pizza night, the students serving dinner put four pizzas on each large table and three pizzas on each small table.



Problem 2.2 More Comparison Strategies

- A. Suppose the pizzas are shared equally by everyone at the table. Does a person sitting at a small table get the same amount as a person sitting at a large table? Explain your reasoning.
- B. Which table relates to $\frac{3}{8}$? What do the 3 and the 8 mean? Is $\frac{3}{8}$ a part-to-whole comparison or a part-to-part comparison?
- C. Selena thinks she can decide at which table a person gets the most pizza. She uses the following reasoning:
- $$10 - 4 = 6 \text{ and } 8 - 3 = 5 \text{ so the large table is better.}$$
1. What does the 6 mean and what does the 5 mean in Selena's method of reasoning?
 2. Do you agree or disagree with Selena's method?
 3. Suppose you put nine pizzas on the large table. What answer does Selena's method give? Does this answer make sense?
 4. What can you now say about Selena's method?



- D. 1.** The ratio of large tables to small tables in the dining room is 8 to 5. There are exactly enough seats for the 240 campers. How many tables of each kind are there?
- 2.** What fraction of the campers sit at small tables?
- 3.** What percent of the campers sit at large tables?

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2.3 Finding Equivalent Ratios

It is often helpful, when forming ratios, to replace the actual numbers being compared with simpler numbers that have the same relationship to each other.

- People prefer Bolda Cola over Cola Nola by a ratio of 17,139 to 11,426, or 3 to 2.
- Students prefer television to radio by a ratio of 100 to 50, or 2 to 1.
- Monthly sales of *Reader's Digest* magazine exceed those of *National Geographic* by 11,044,694 to 6,602,650, or about 3 to 2.

Getting Ready for Problem 2.3

Suppose all classes at your grade level took the cola taste test. The result was 100 to 80 in favor of Bolda Cola.

- How do you scale down this ratio to make it easier to understand?
- What are some other ratios equivalent to this ratio in which the numbers are greater? Finding greater numbers is scaling *up* the ratio.
- How is scaling ratios like finding equivalent fractions for $\frac{100}{80}$? How is it different?

Problem 2.3 Scaling Ratios

One of Ming's tasks at the county zoo's primate house is to mix food for the chimpanzees. The combination of high-fiber nuggets and high-protein nuggets changes as the chimps grow from babies to adults.

Ming has formulas for mixing high-fiber and high-protein nuggets for the chimps.

- Baby chimps: 2 cups high-fiber nuggets and 3 cups high-protein nuggets per serving
- Young adult chimps: 6 cups high-fiber nuggets and 4 cups high-protein nuggets per serving
- Older chimps: 4 cups high-fiber nuggets and 2 cups high-protein nuggets per serving

- A. 1.** What amounts of high-fiber and high-protein nuggets will Ming need when she has to feed 2 baby chimps? 3 baby chimps? 4 baby chimps?

Copy and complete the table below.

Dietary Needs of Baby Chimps

Number of Baby Chimps	1	2	3	4	5	10
Cups of High-Fiber Nuggets	■	■	■	■	■	■
Cups of High-Protein Nuggets	■	■	■	■	■	■

- 2.** What patterns do you see in your table?
- 3.** Ming puts 48 cups of high-protein nuggets into the baby chimp mix. How many cups of high-fiber nuggets does she put into the mix? Explain.
- 4.** Ming has a total of 125 cups of mix for baby chimps. How many cups of high-fiber nuggets are in the mix? Explain.
- B. 1.** What is the ratio of high-fiber to high-protein nuggets for young adult chimps?
- 2.** Scale this ratio up to show the ratio of high-fiber to high-protein nuggets that will feed 21 young adult chimps.
- 3.** To feed 18 young adults, you need 108 cups of high-fiber nuggets and 72 cups of high-protein nuggets. Show how to scale down this ratio to feed 3 young adult chimps.

- C. 1. Darla wants to compare the amount of high-fiber nuggets to the total amount of food mix for young adult chimps. She makes this claim:

“High-fiber nuggets are $\frac{3}{2}$ of the total.”

Lamar says Darla is wrong. He makes this claim:

“High-fiber nuggets are $\frac{3}{5}$ of the total.”

Who is correct? Explain.

2. What fraction of the total amount of food mix for older chimps is high-fiber nuggets?
3. Suppose the ratio of male chimps to female chimps in a zoo is 5 to 4. What fraction of the chimps are male?
4. Suppose $\frac{2}{3}$ of the chimps in a zoo are female. Find the ratio of female chimps to male chimps in that zoo.



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Applications

As you work on the ACE exercises, try a variety of reasoning methods. Then think about conditions when each method seems most helpful.

1. Compare these four mixes for apple juice.

Mix W	
5 cups	8 cups
concentrate	cold water

Mix X	
3 cups	6 cups
concentrate	cold water

Mix Y	
6 cups	9 cups
concentrate	cold water

Mix Z	
3 cups	5 cups
concentrate	cold water


- a. Which mix would make the most “appley” juice?
 - b. Suppose you make a single batch of each mix. What fraction of each batch is concentrate?
 - c. Rewrite your answers to part (b) as percents.
 - d. Suppose you make only 1 cup of Mix W. How much water and how much concentrate do you need?
2. Examine these statements about the apple juice mixes in Exercise 1. Decide whether each is accurate. Give reasons for your answers.
 - a. Mix Y has the most water, so it will taste least “appley.”
 - b. Mix Z is the most “appley” because the difference between the concentrate and water is 2 cups. It is 3 cups for each of the others.
 - c. Mix Y is the most “appley” because it has only $1\frac{1}{2}$ cups of water for each cup of concentrate. The others have more water per cup.
 - d. Mix X and Mix Y taste the same because you just add 3 cups of concentrate and 3 cups of water to turn Mix X into Mix Y.

3. If possible, change each comparison of concentrate to water into a ratio. If not possible, explain why.
- The mix is 60% concentrate.
 - The fraction of the mix that is water is $\frac{3}{5}$.
 - The difference between the amount of concentrate and water is 4 cups.
4. At camp, Miriam uses a pottery wheel to make three bowls in 2 hours. Duane makes five bowls in 3 hours.
- Who makes bowls faster, Miriam or Duane?
 - At the same pace, how long will it take Miriam to make a set of 12 bowls?
 - At the same pace, how long will it take Duane to make a set of 12 bowls?
5. Guests at a pizza party are seated at 3 tables. The small table has 5 seats and 2 pizzas. The medium table has 7 seats and 3 pizzas. The large table has 12 seats and 5 pizzas. The pizzas at each table are shared equally. At which table does a guest get the most pizza?



6. For each business day, news reports tell the number of stocks that gained (went up in price) and the number that declined (went down in price). In each of the following pairs of reports, determine which is better news for investors.

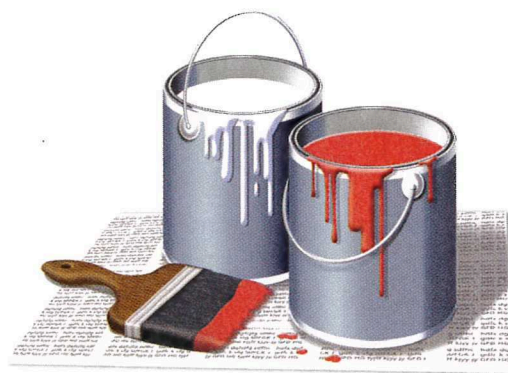
- a. $\left[\begin{array}{l} \text{Gains outnumber declines} \\ \text{by a ratio of 5 to 3.} \end{array} \right]$ OR $\left[\begin{array}{l} \text{Gains outnumber declines} \\ \text{by a ratio of 7 to 5.} \end{array} \right]$
- b. $\left[\begin{array}{l} \text{Gains outnumber declines} \\ \text{by a ratio of 9 to 5.} \end{array} \right]$ OR $\left[\begin{array}{l} \text{Gains outnumber declines} \\ \text{by a ratio of 6 to 3.} \end{array} \right]$
- c. $\left[\begin{array}{l} \text{Declines outnumber gains} \\ \text{by a ratio of 10 to 7.} \end{array} \right]$ OR $\left[\begin{array}{l} \text{Declines outnumber gains} \\ \text{by a ratio of 6 to 4.} \end{array} \right]$

Homework
Help  **Online**
PHSchool.com
For: Help with Exercise 6
Web Code: ane-3206

7. Suppose a news story about the Super Bowl claims “Men outnumbered women in the stadium by a ratio of 9 to 5.” Does this mean that there were 14 people in the stadium—9 men and 5 women? If not, what does the statement mean?
8. **Multiple Choice** Which of the following is a correct interpretation of the statement “Men outnumbered women by a ratio of 9 to 5?”
- A. There were four more men than women.
 - B. The number of men was 1.8 times the number of women.
 - C. The number of men divided by the number of women was equal to the quotient of $5 \div 9$.
 - D. In the stadium, five out of nine fans were women.

Connections

9. If possible, change each comparison of red paint to white paint to a percent comparison. If it is not possible, explain why.
- a. The fraction of a mix that is red paint is $\frac{1}{4}$.
 - b. The ratio of red to white paint in a different mix is 2 to 5.
10. If possible, change each comparison to a fraction comparison. If it is not possible, explain why.
- a. The nut mix has 30% peanuts.
 - b. The ratio of almonds to other nuts in the mix is 1 to 7.
11. Find a value that makes each sentence correct.
- a. $\frac{3}{15} = \frac{\blacksquare}{30}$
 - b. $\frac{1}{2} < \frac{\blacksquare}{20}$
 - c. $\frac{\blacksquare}{20} > \frac{3}{5}$
 - d. $\frac{9}{30} \leq \frac{\blacksquare}{15}$
 - e. $\frac{\blacksquare}{12} \geq \frac{3}{4}$
 - f. $\frac{9}{21} = \frac{12}{\blacksquare}$



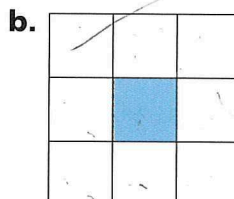
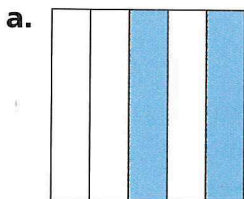
12. Use the table to answer parts (a)–(e).

Participation in Walking for Exercise

	Ages 12–17	Ages 55–64
People Who Walk	3,781,000	8,694,000
Total in Group	23,241,000	22,662,000

SOURCE: U.S. Census Bureau. Go to PHSchool.com for a data update. Web Code: ang-9041

- a. What percent of the 55–64 age group walk for exercise?
 b. What percent of the 12–17 age group walk for exercise?
 c. Write a ratio statement to compare the number of 12- to 17-year-olds who walk to the number of 55- to 64-year-olds who walk. Use approximate numbers to simplify the ratio.
 d. Write a ratio statement to compare the percent of 12- to 17-year-olds who walk for exercise to the percent of 55- to 64-year-olds who walk for exercise.
 e. Which data—actual numbers of walkers or percents—would you use in comparing the popularity of exercise walking among various groups? Explain.
13. The probability of getting a sum of 5 when you roll two number cubes is $\frac{4}{36}$. How many times should you expect to get a sum of 5 if you roll the cubes each number of times?
 a. 9 b. 18 c. 27 d. 100 e. 450
14. For each diagram, write three statements comparing the areas of the shaded and unshaded regions. In one statement, use fraction ideas to express the comparison. In the second, use percent ideas. In the third, use ratio ideas.



15. **Multiple Choice** Choose the value that makes $\frac{18}{30} = \frac{\square}{15}$ correct.
 F. 7 G. 8 H. 9 J. 10
16. **Multiple Choice** Choose the value that makes $\frac{\square}{15} \leq \frac{3}{5}$ correct.
 A. 9 B. 10 C. 11 D. 12

17. Find a value that makes each sentence correct. Explain your reasoning in each case.

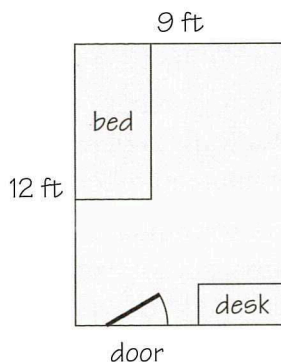
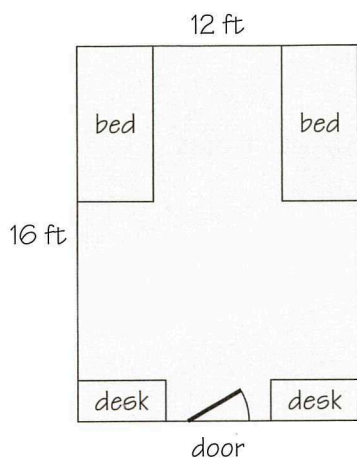
a. $\frac{3}{4} = \frac{\blacksquare}{12}$

b. $\frac{3}{4} < \frac{\blacksquare}{12}$

c. $\frac{3}{4} > \frac{\blacksquare}{12}$

d. $\frac{9}{12} = \frac{12}{\blacksquare}$

18. The sketches show floor plans for dorm rooms for two students and for one student.



- a. Are the floor plans similar rectangles? If so, what is the scale factor? If not, why not?
- b. What is the ratio of floor areas of the two rooms (including space under the beds and desks)?
- c. Which type of room gives more space per student?
19. Find values that make each sentence correct.
- a. $\frac{6}{14} = \frac{\blacksquare}{21} = \frac{\blacksquare}{28}$
- b. $\frac{\blacksquare}{27} = \frac{8}{36} = \frac{\blacksquare}{63}$
- c. $\frac{\blacksquare}{20} = \frac{\blacksquare}{25} = \frac{6}{30}$
- d. $\frac{\blacksquare}{8} = \frac{15}{\blacksquare} = \frac{24}{32}$
20. Suppose a news story reports, "90% of the people in the Super Bowl stadium were between the ages of 25 and 55." Alicia thinks this means only 100 people were in the stadium, and 90 of them were between 25 and 55 years of age. Do you agree with her? If not, what does the statement mean?
21. Suppose a news story reports, "A survey found that $\frac{4}{7}$ of all Americans watched the Super Bowl on television." Bishnu thinks this means the survey reached seven people and four of them watched the Super Bowl on television. Do you agree with him? If not, what does the statement mean?

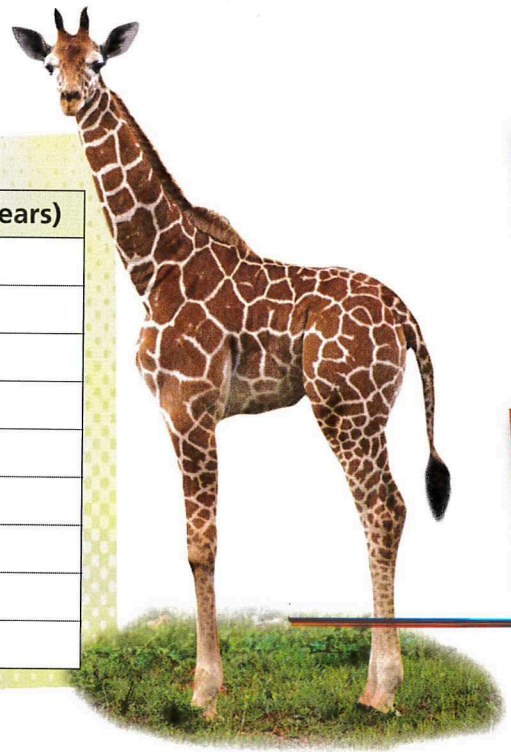
Extensions

22. Mammals vary in the length of their pregnancies, or gestations. *Gestation* is the time from conception to birth. Use the table to answer the questions that follow.

Gestation Times and Life Spans of Selected Mammals

Animal	Gestation (days)	Life Span (years)
Chipmunk	31	6
Cat	63	12
Fox	52	7
Lion	100	15
Black Bear	219	18
Gorilla	258	20
Moose	240	12
Giraffe	425	10
Elephant (African)	660	35

SOURCE: *The World Almanac and Book of Facts*



- Plan a way to compare life span and gestation time for animals and use it with the data.
- Which animal has the greatest ratio of life span to gestation time? Which has the least ratio?
- Plot the data on a coordinate graph using (*gestation, life span*) as data points. Describe any interesting patterns that you see. Decide whether there is any relation between the two variables. Explain how you reached your conclusion.
- What pattern would you expect to see in a graph if each statement were true?
 - Longer gestation time implies longer life span.
 - Longer gestation time implies shorter life span.



- 23.** The city of Spartanville runs two summer camps—the Green Center and the Blue Center. The table below shows recent attendance at the two camps.

	Green	Blue
Boys	125	70
Girls	75	30

In this exercise, you will show how several approaches can be used to answer the following question.

- Which center seems to offer a camping program that appeals best to girls?
- What conclusion would you draw if you focused on the differences between the numbers of boy and girl campers from each center?
 - How could you use fractions to compare the appeal of the two centers' camping programs for boys and girls? What conclusion would you draw?
 - How could you use percents to compare the appeal of the two centers' camping programs for boys and girls? What conclusion would you draw?
 - How could you use ratios to compare the appeal of the two centers' camping programs for boys and girls? What conclusion would you draw?

24. Use the table below.

**Participation in Team Sports
at Springbrook Middle School**

Sport	Girls	Boys
Basketball	30	80
Football	10	60
Soccer	120	85
Total Surveyed	160	225

- a. In which sport do boys most outnumber girls?
 - b. In which sport do girls most outnumber boys?
 - c. The participation in these team sports is about the same for students at Key Middle School.
 - i. Suppose 250 boys at Key play sports. How many would you expect to play each of the three sports?
 - ii. Suppose 240 girls at Key play sports. How many would you expect to play each of the three sports?
-

Mathematical Reflections

2

In this investigation, you solved problems by comparing ratios, percents, and fractions. You also used ratio, percent, and fraction data to solve problems of larger or smaller scale. The following questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. The director of a recreation center wants to compare the 10 boys to the 20 girls who attend its camping program.
 - a. How would you make a comparison using fractions?
 - b. How would you make a comparison using percents?
 - c. How would you make a comparison using ratios?
 - d. How is your percent comparison related to your ratio comparison?
 - e. How is your fraction comparison related to your percent comparison?
2.
 - a. Explain how you would scale up the ratio 10 boys to 14 girls to find equivalent ratios.
 - b. Explain how you would scale down the ratio 10 boys to 14 girls to find equivalent ratios.